

**PhD Summer School
Formal Methods for System Analysis in Informatics
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High Level Petri Nets

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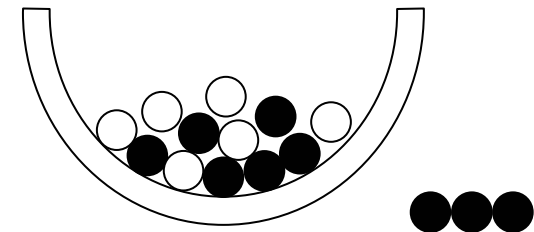
Introducing High Level Petri Nets

Black and white pebble “game”

Given: Pot with m **white** and n **black** pebbles + supply of $\geq \lceil m/2 \rceil$ black pebbles

1 Move: Take any two pebbles out of the pot.

- $B + W \rightarrow$ put 1 W back.
- $W + W \rightarrow$ put 1 B into pot.
- $B + B \rightarrow$ put 1 B back.

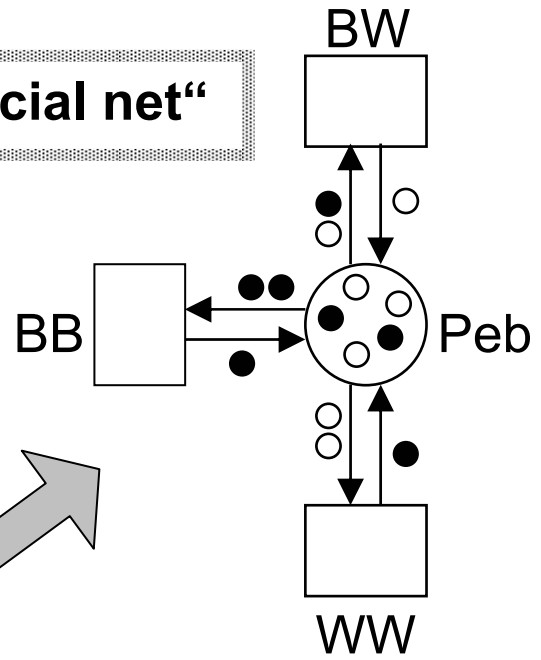


Questions: Initial situation $(m,n) \rightarrow$ **one** or **several** possible final situations?
If exactly one, $final(m,n) \in \mathbf{N}_0 \times \mathbf{N}_0$, **calculate** it!

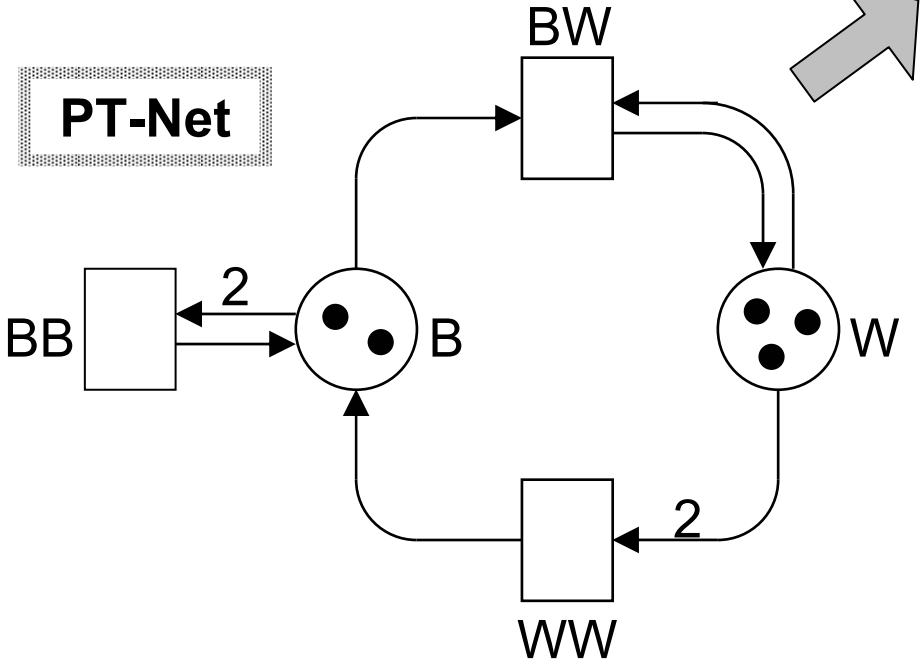
Task: **Model** the system's behaviour and states.

Pebble game nets

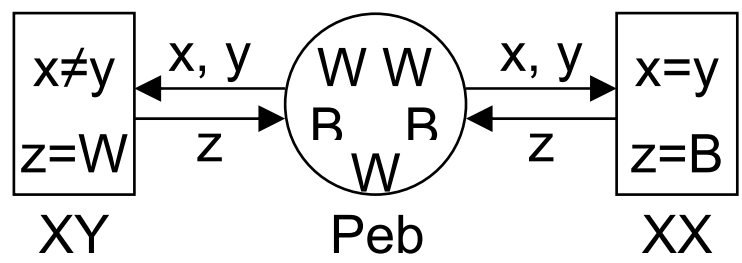
„special net“



PT-Net

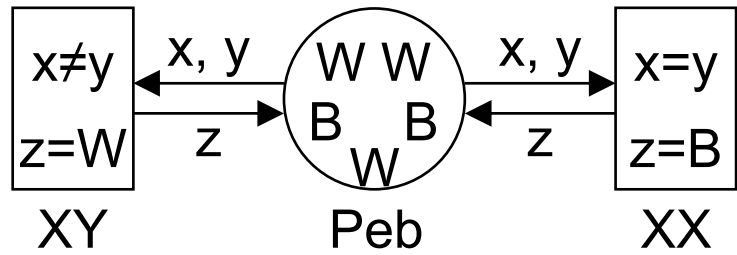


Basic High Level Petri net
BHLPN

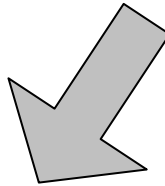


More pebble game nets

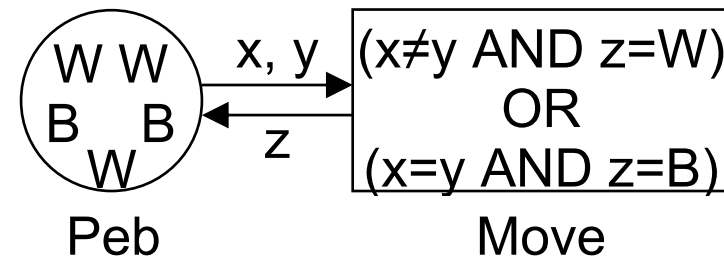
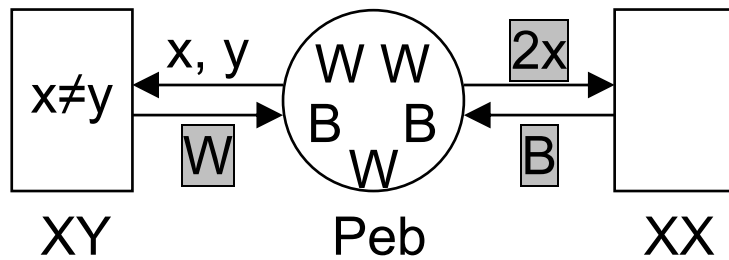
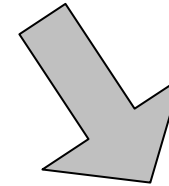
**Basic High Level Petri net
BHLPN**



Notation:
expressions on arcs,
equality of tokens



**Max.
Folding**



Informal definition of High Level Petri Nets

PT systems:

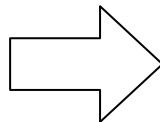
- tokens are **all alike** and cannot be distinguished

High level Petri nets:

- tokens are individual; they can be **distinguished**;
- one can **calculate** with them.

For various reasons
stylistic preferences,
various dis/advantages of various variants,
publish or perish,
many kinds of HLPNs have been defined.

Formal definitions of HLPNs are often very long.



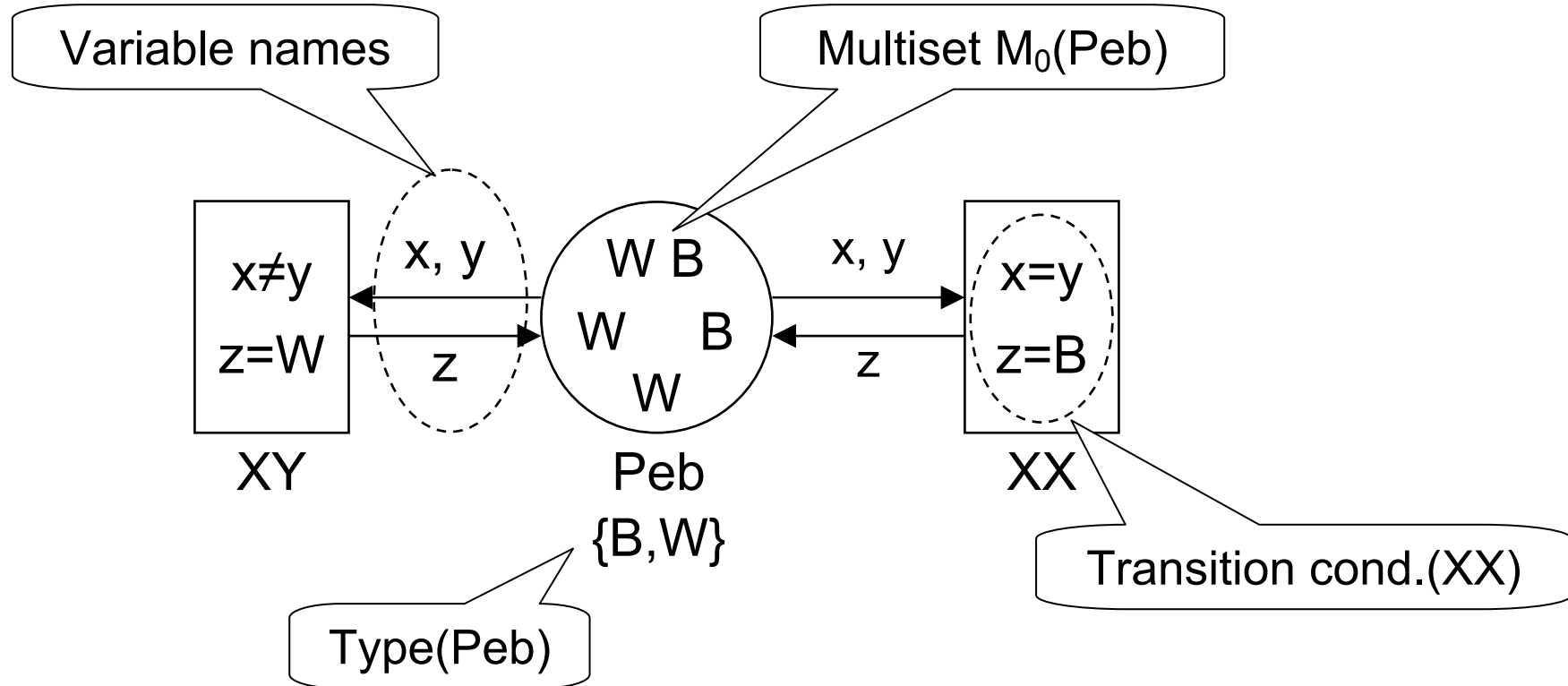
For the sake of simplicity, we introduce **informally** what we call **Basic HLPNs**.

Basic HLPNs

... differ from PT systems in the the following static features:

- Each **place** \mapsto a data type $Type(p)$
(token sort, class, kind).
- Each **marking** of a place = a multiset of individual tokens
(of the type assigned to the place).
- Each **arc weight** of an arc = a list of variable names
all different around any transition
(can be referred to in the
transition conditions).
- Each **transition** \mapsto a transition condition $Cond(t)$
(requirements on input and output
tokens).

BHLPN features



HLPN separate declaration part

- declare **concrete data types**
(sorts, constants, operations, properties, relations),
e.g. as provided by a **programming language** or a dedicated
declaration part language
- or declare an **abstract data type**
in some **ADT dialect**
- assign token **types/sorts** to **places** of the net

Dynamic features of Basic HLPNs

The **occurrence** of a transition t is determined by an

- **assignment** ass of values to the variables appearing in the arc weights around t .

transition occurrence = (transition name, assignment)

This assignment associates

- to each arc weight $W(f)$ (= variable list) concerned
- a **multiset** of values $ass_w(W(f))$.

The occurrence of t with this assignment must fulfil the following conditions:

- For each input arc (p,t) , the multiset $ass_w(W(p,t))$ must be contained in the given marking $M(p)$ of the input place.
- The assignment must make $ass_C(Cond(t))$, the transition condition of t , true.

enough
tokens?

with the right
properties?

The occurrence(s) of an HLPN transition

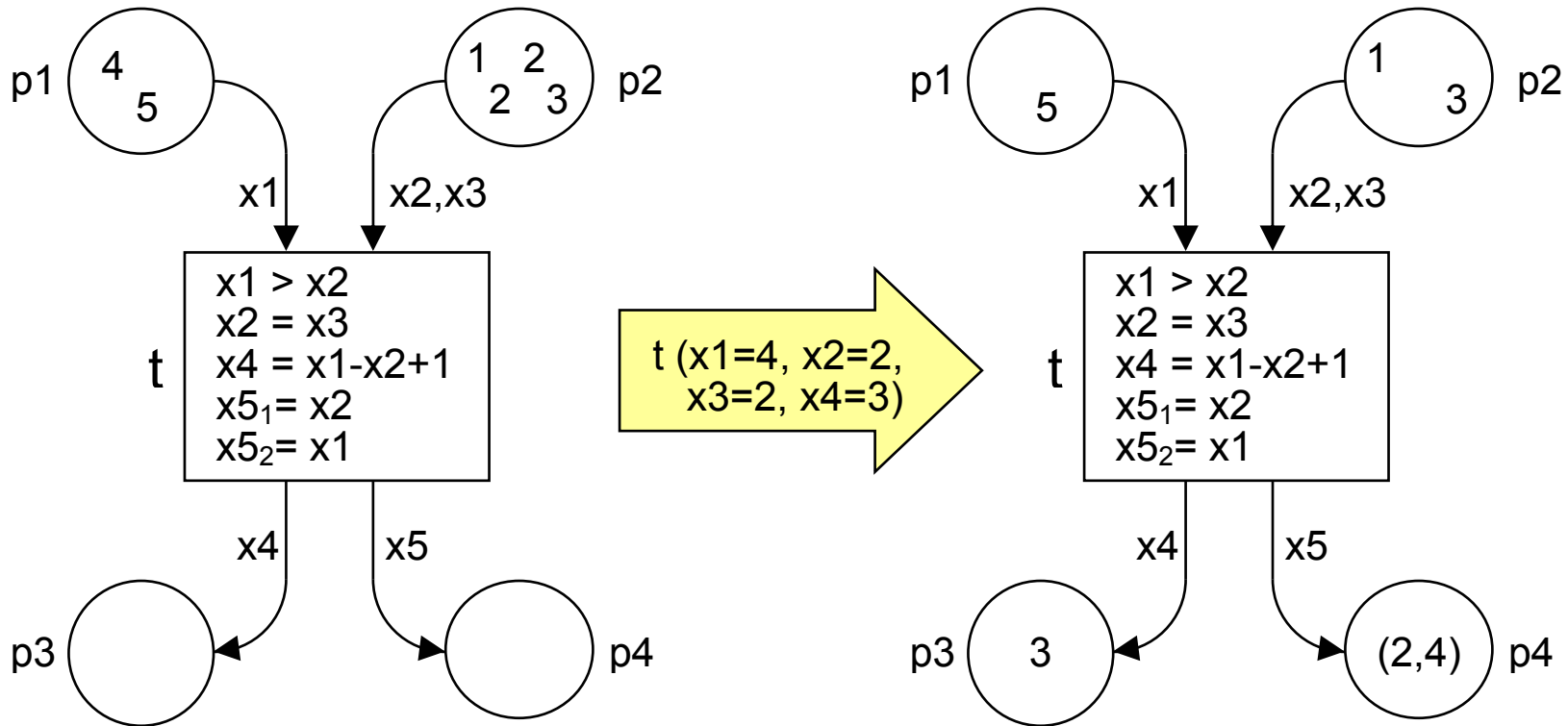
This occurrence of t has the following effects:

- For each input arc (p,t) ,
the multiset $\mathbf{ass}(W(p,t))$ is **taken away** (subtracted) from $M(p)$.
- For each output arc (t,p) ,
the multiset $\mathbf{ass}(W(t,p))$ is **added** to (whatever is left of) $M(p)$.

All this happens atomically, without (noticeable) intermediate state.

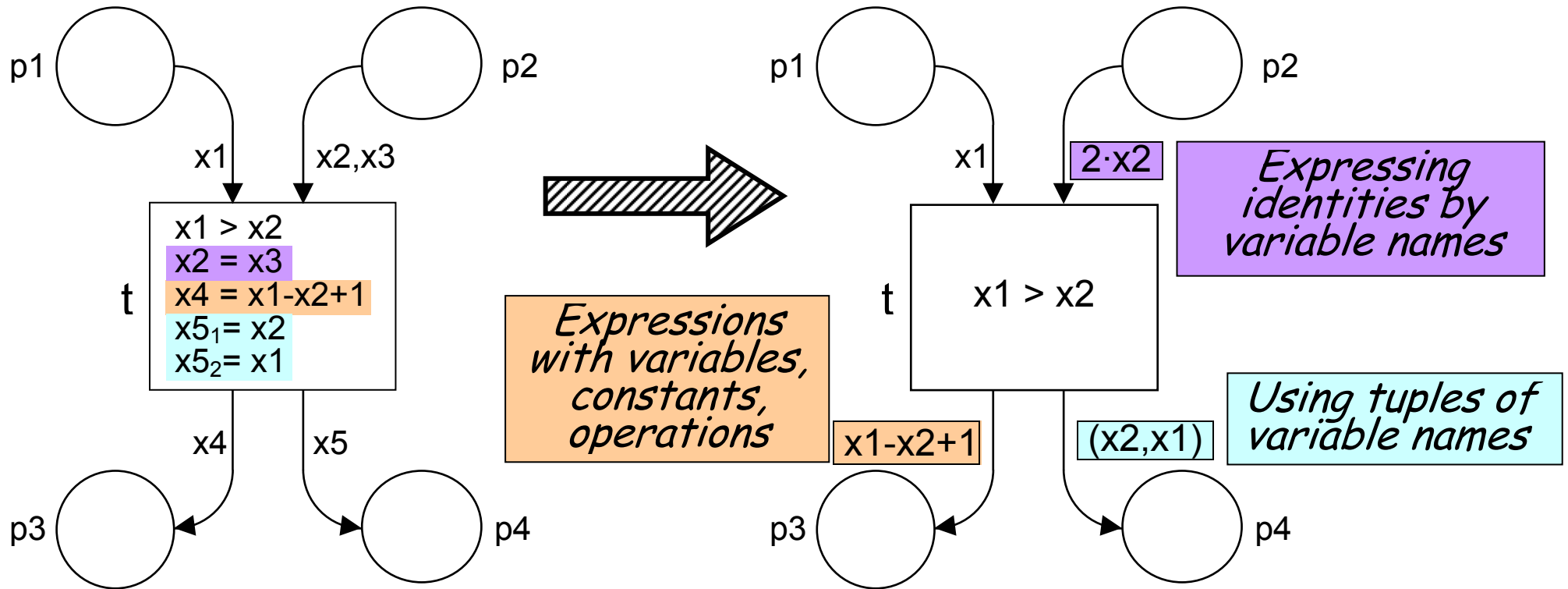
The occurrence(s) of an HLPN transition, example

p1, p2, p3 are of type IN. p4 is of type IN x IN.



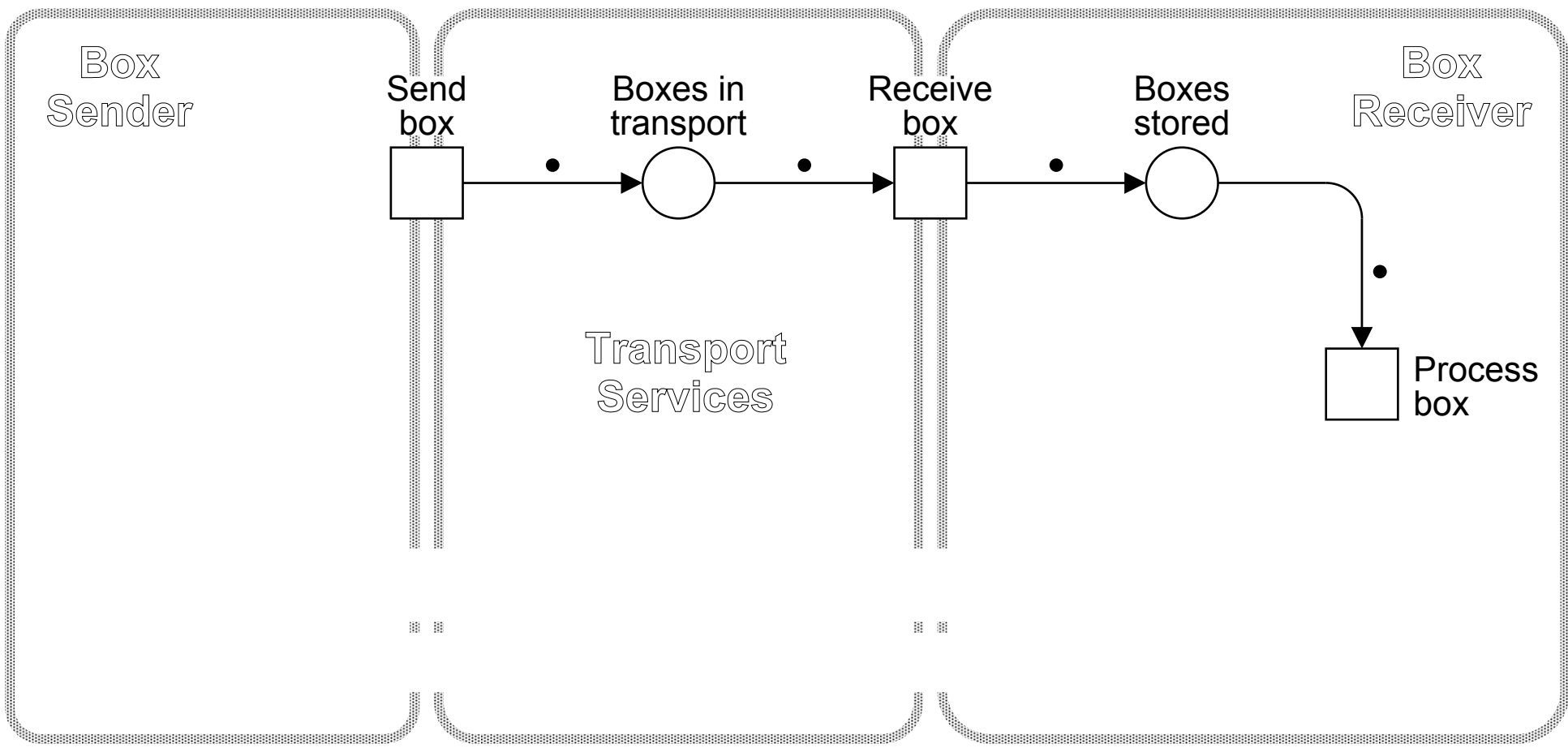
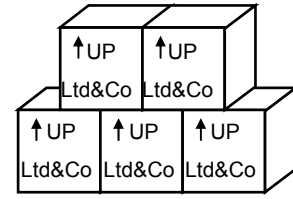
t above can also fire a **different occurrence with different** parameter (occurrence variable) **values**.

Notational simplifications:

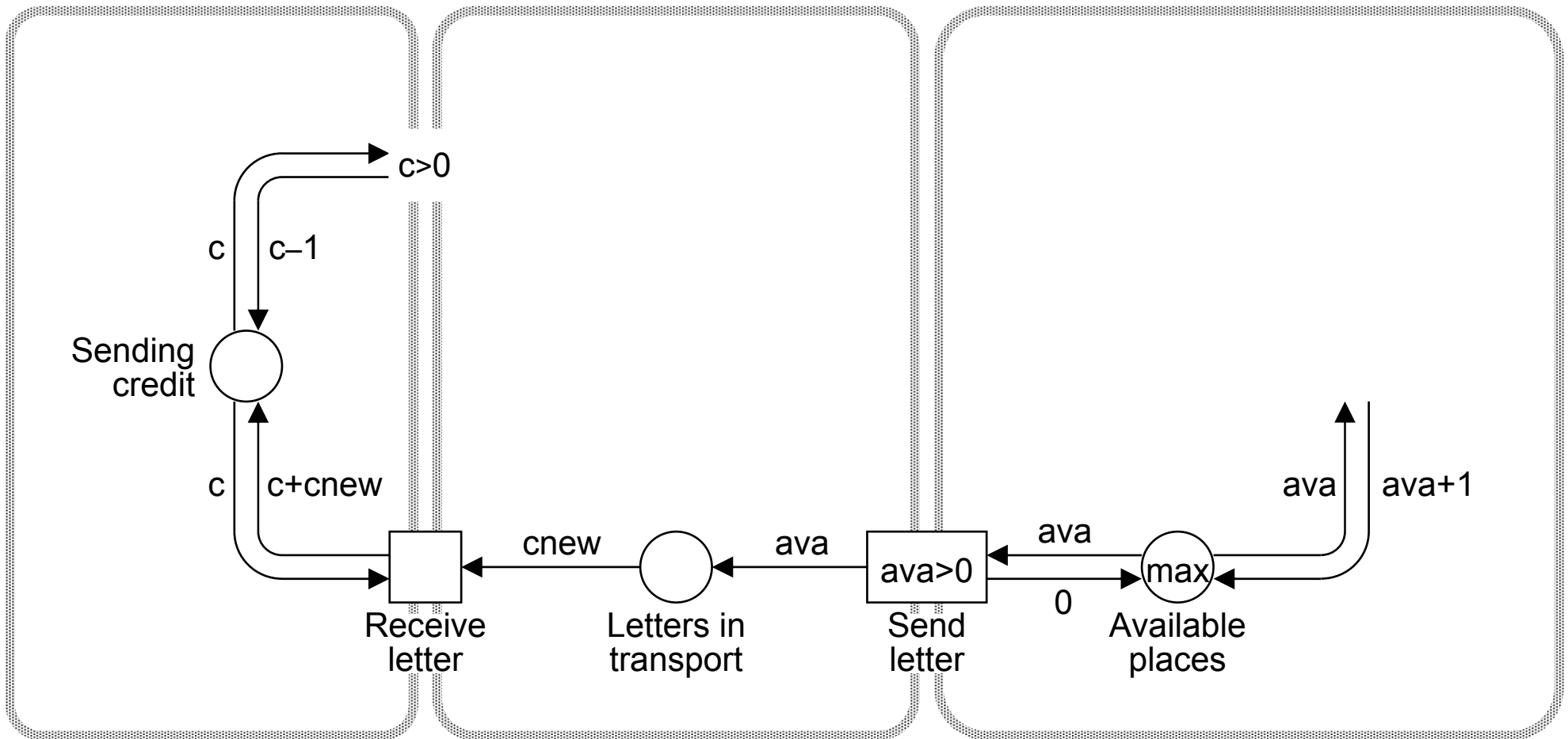


Working with HLPNs: a flow control example

A flow-control **problem**: limited storage space for boxes but reception cannot be blocked



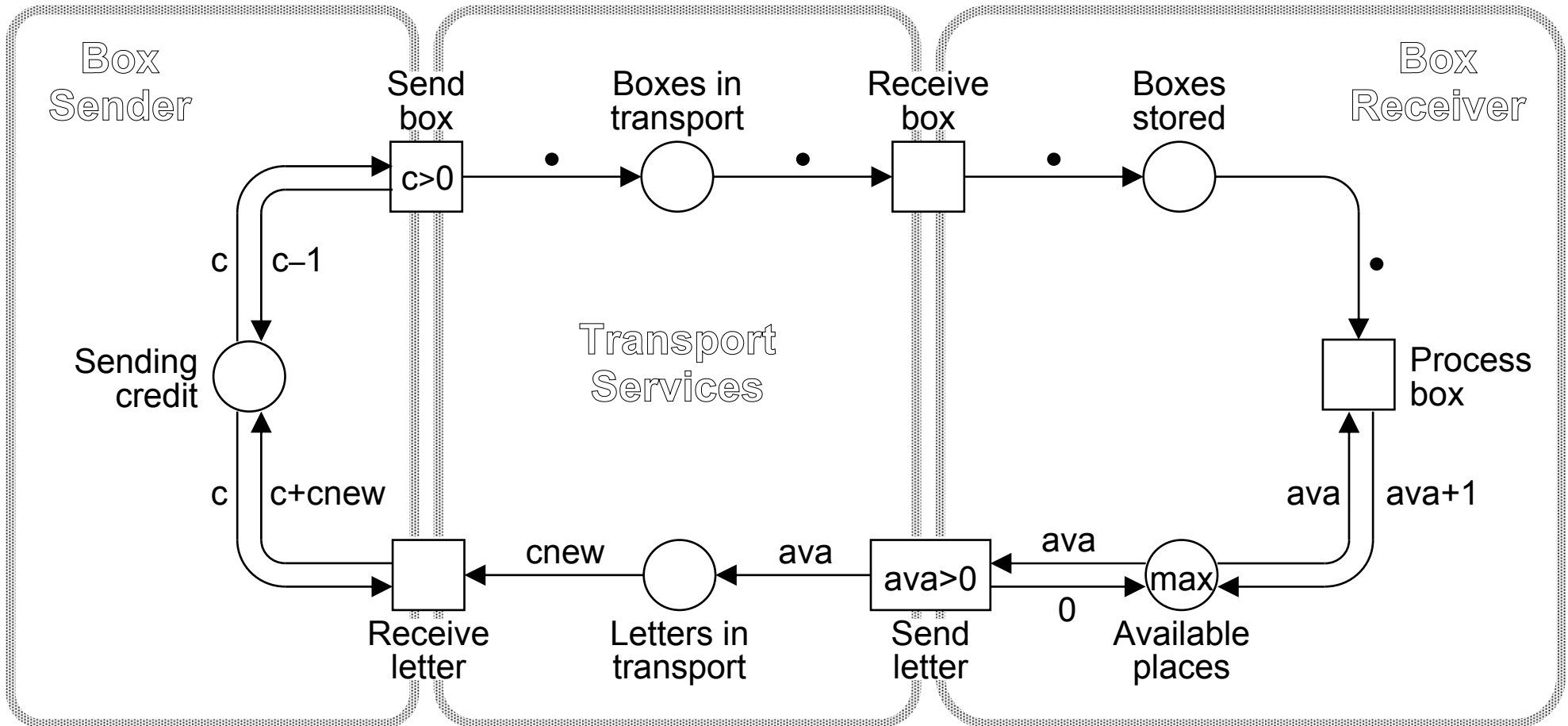
Working with HLPNs: a flow control example



A flow-control **solution**: the “boxes and letters protocol”



Correctness \leftrightarrow an invariant w.r.t. transition occurrences



$$|nr \text{ in } ava \text{ pl}| + \sum(nrs \text{ in } letters) + |nr \text{ in } send \text{ cred}| + \# \text{ tokens in boxes in } tp + \# \text{tokens in boxes stored} = \text{constant} = \text{max}$$